SKMM 3033
Finite Element Method

Topic 6: Coordinate transformation for Truss stiffness matrix

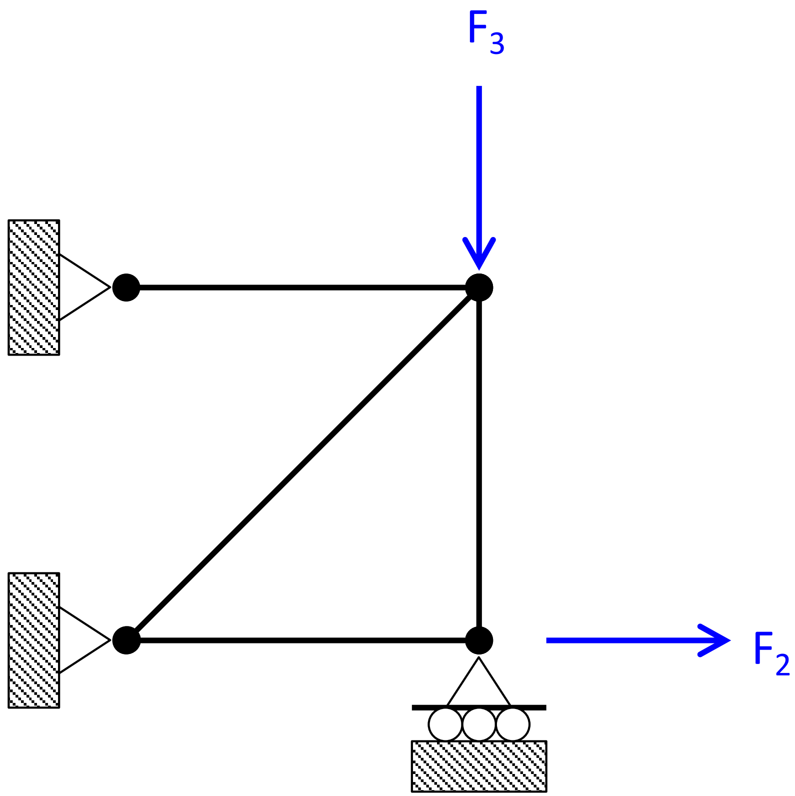
DR WILLIAM CHONG WOEI FONG
Department of Aeronautics, Automotive and Ocean Engineering
Faculty of Mechanical Engineering

By the end of the notes:

The students are expected:

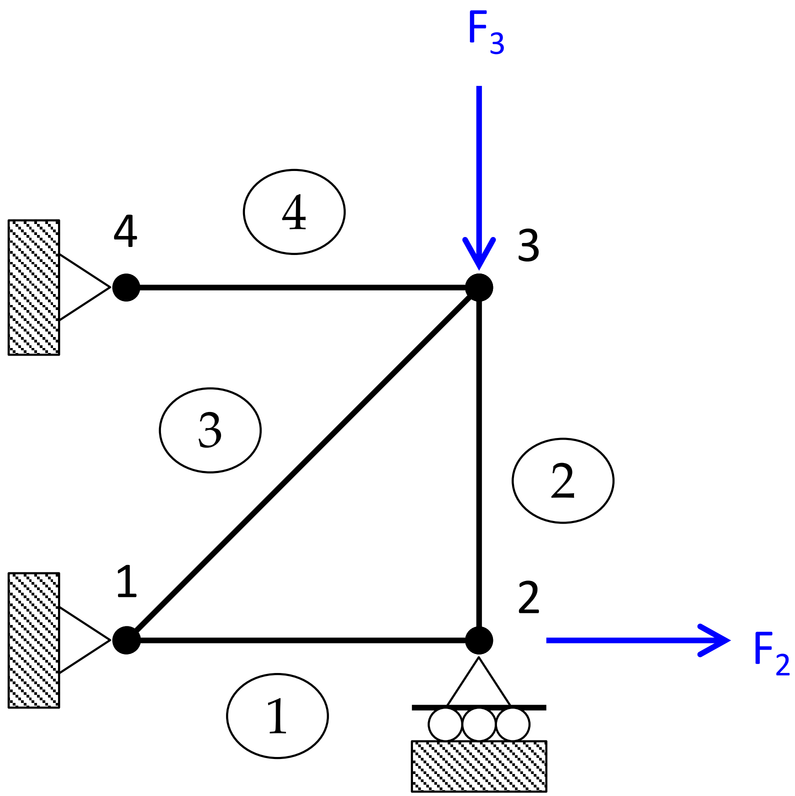
- To execute coordinate transformations of 1-dimensional bar element for plane trusses
- To derive element stiffness matrix for plane trusses

Plane Trusses



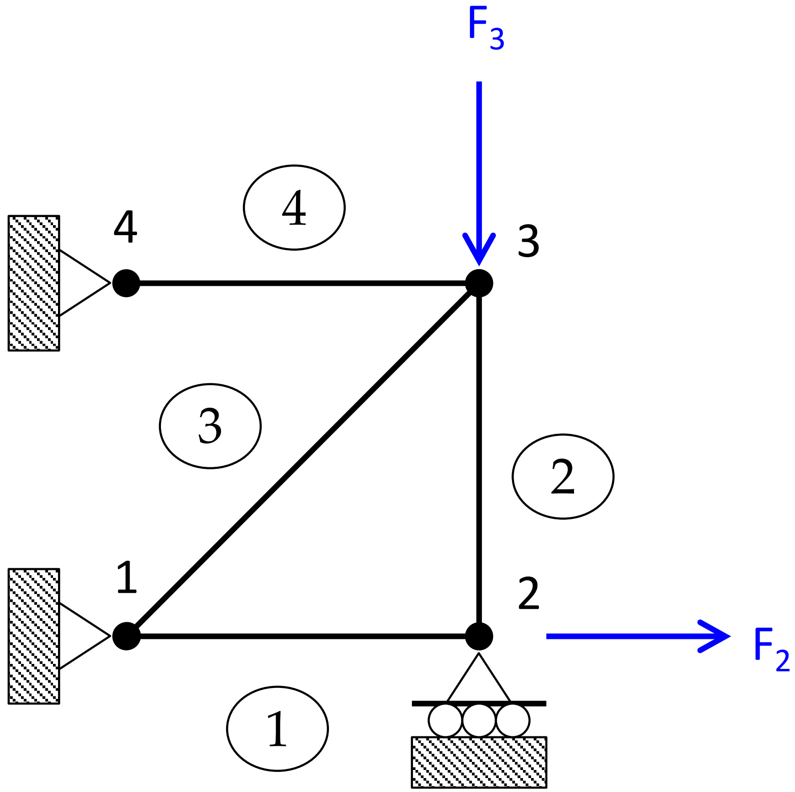
- A typical two-dimensional plane truss comprises of two-force members
- It is connected by frictionless joints.
- All loads and reaction forces are applied at the joints only.

Plane Trusses



- A typical two-dimensional plane truss comprises of two-force members
- It is connected by frictionless joints.
- All loads and reaction forces are applied at the joints only.

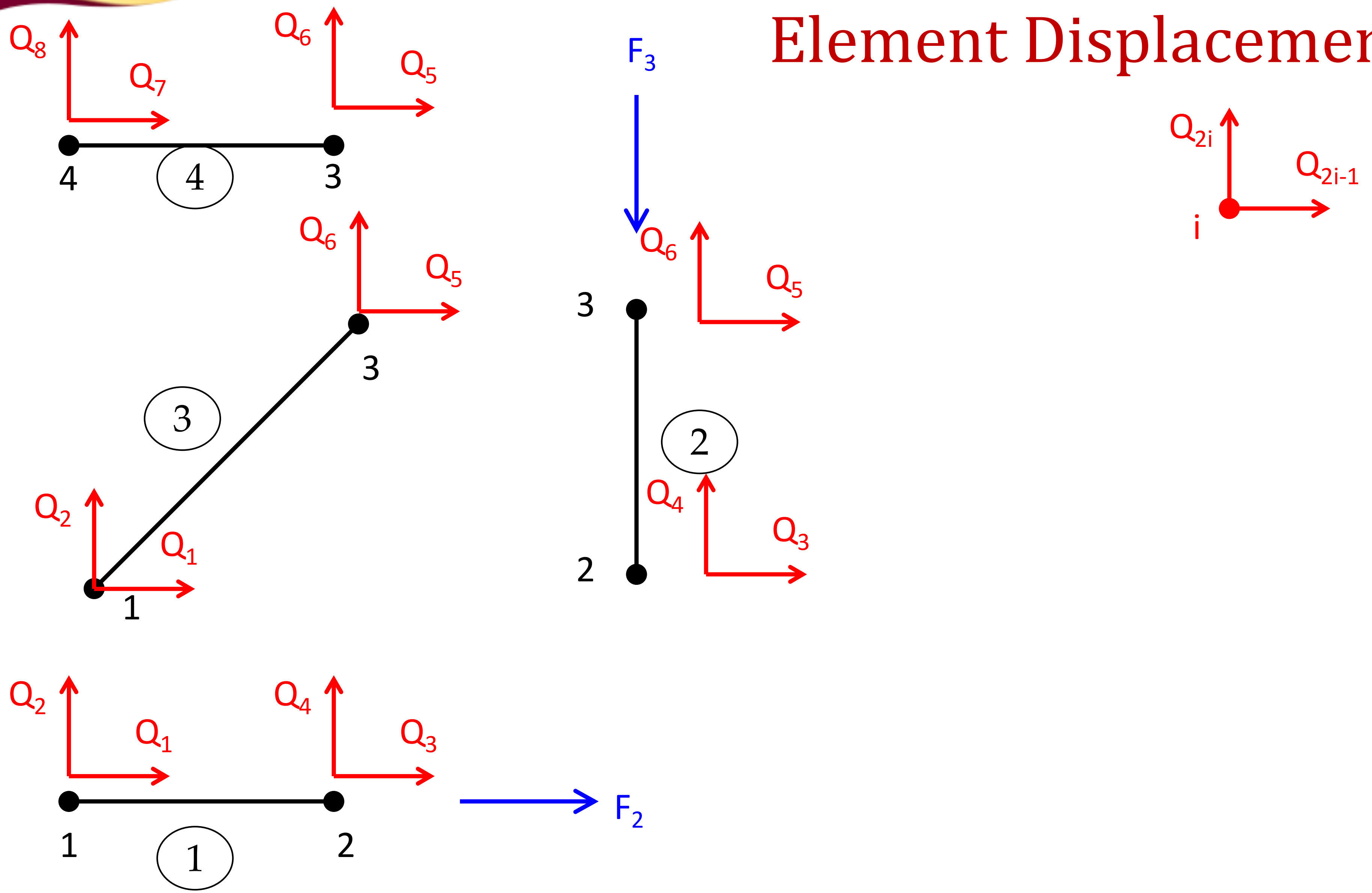
Elements & Nodes



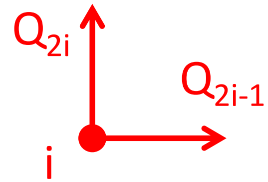
Element Connectivity Table

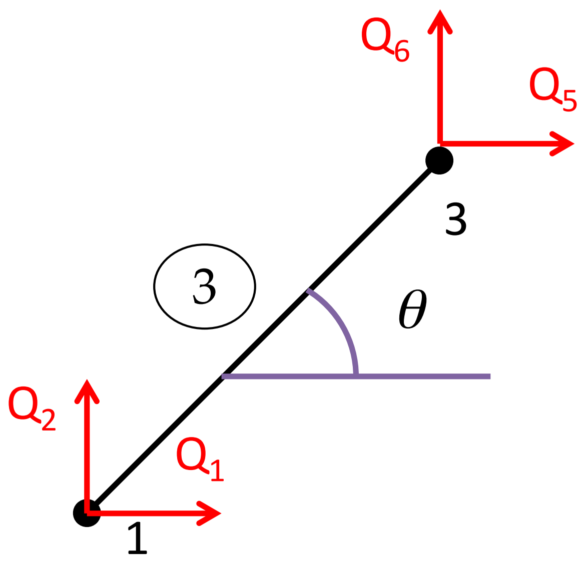
Elements	Nodes	
1	1	2
2	3	2
3	1	3
4	4	3

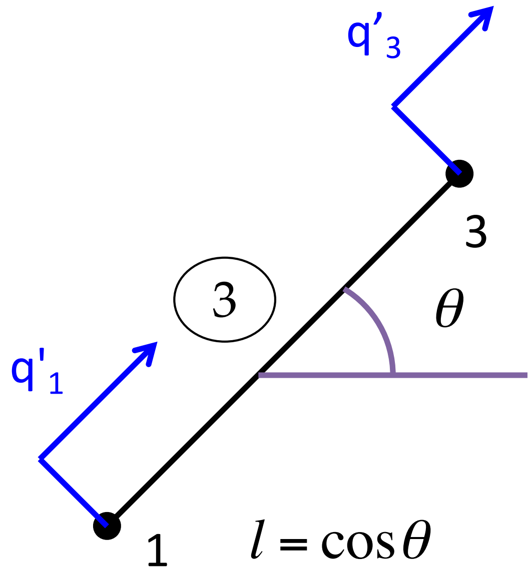
Element Displacement



Element Displacement







$$l = \cos\theta$$

$$m = \sin\theta$$

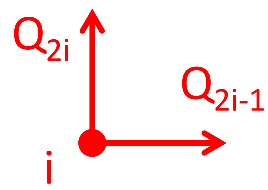
$$\{q\} = \begin{pmatrix} q_1 \\ q_2 \\ q_5 \\ q_6 \end{pmatrix}$$

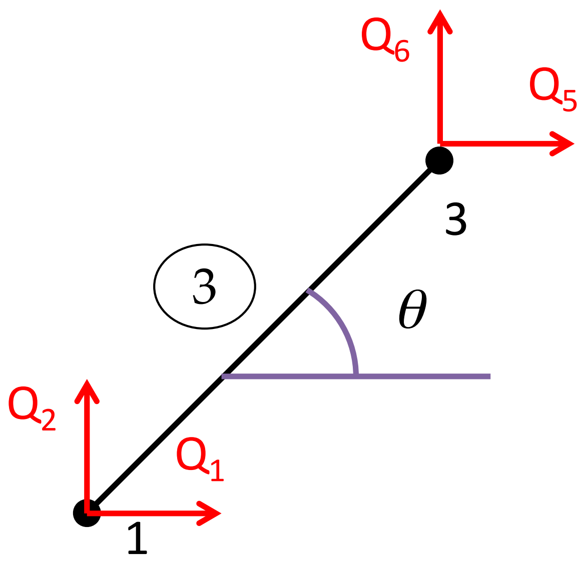
$$\{q'\} = \begin{pmatrix} q'_1 \\ q'_3 \end{pmatrix}$$

$$q'_1 = q_1\cos\theta + q_2\sin\theta$$

$$q'_5 = q_5\cos\theta + q_6\sin\theta$$

Element Displacement



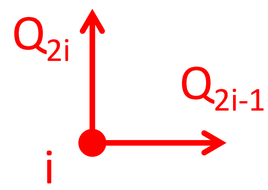


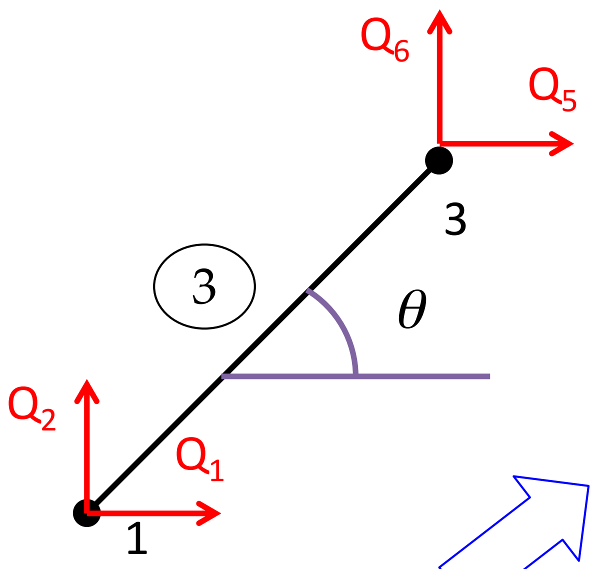
$$\{q\} = \begin{pmatrix} q_1 \\ q_2 \\ q_5 \\ q_6 \end{pmatrix}$$

$$q'_1 = q_1 l + q_2 m$$
$$q'_3 = q_5 l + q_6 m$$

$$q'_1 = q_1 \cos\theta + q_2 \sin\theta$$
$$q'_3 = q_5 \cos\theta + q_6 \sin\theta$$

Element Displacement



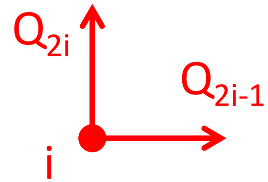


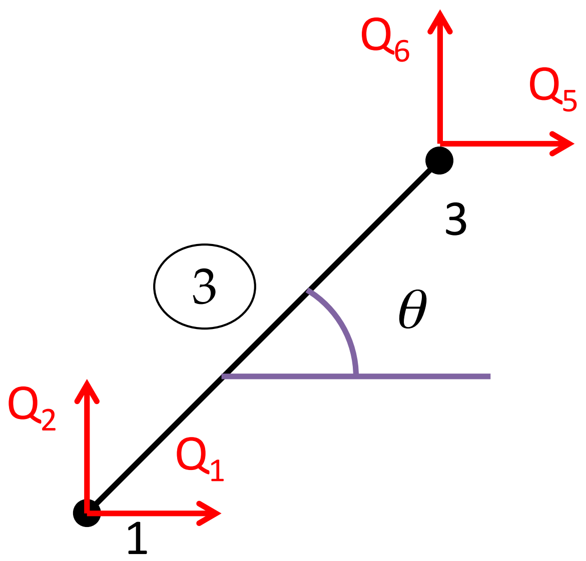
$$\therefore \begin{pmatrix} q'_1 \\ q'_3 \end{pmatrix} = \begin{pmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_5 \\ q_6 \end{pmatrix}$$

$$\{q\} = \begin{pmatrix} q_1 \\ q_2 \\ q_5 \\ q_6 \end{pmatrix}$$

$$q'_1 = q_1 l + q_2 m$$
$$q'_3 = q_5 l + q_6 m$$

Element Displacement

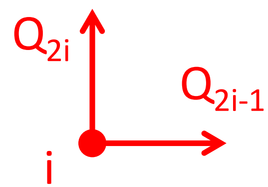


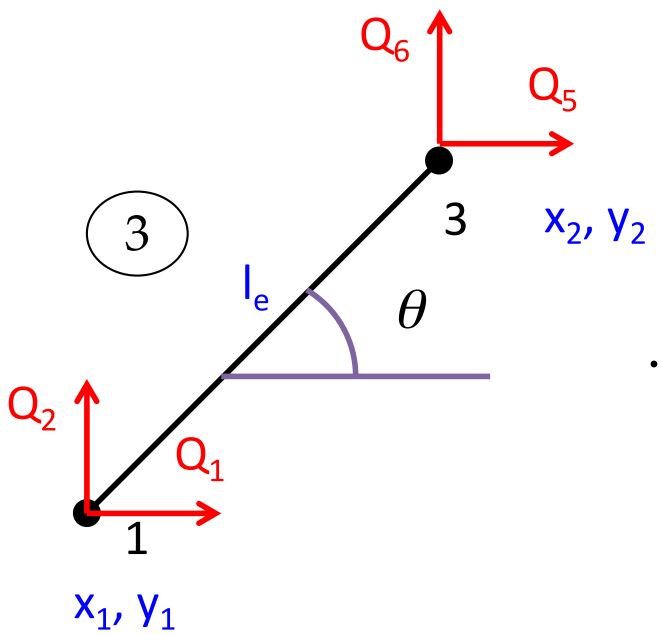


$$\therefore \begin{pmatrix} q'_1 \\ q'_3 \end{pmatrix} = \begin{pmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_5 \\ q_6 \end{pmatrix}$$

$$\{q'\} = \{L\}\{q\}$$

Element Displacement





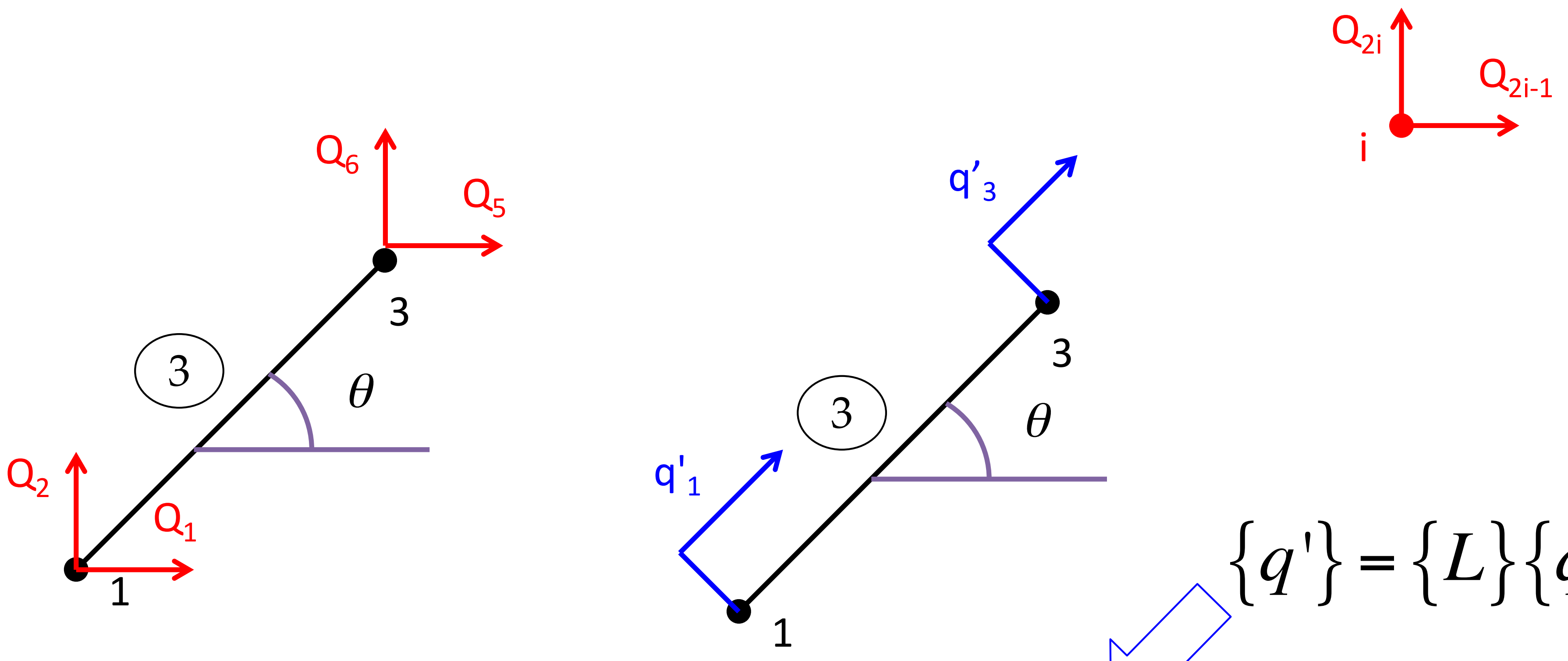
$$\therefore \begin{pmatrix} q'_1 \\ q'_3 \end{pmatrix} = \begin{pmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_5 \\ q_6 \end{pmatrix}$$

$$l = \cos\theta = \frac{x_2 - x_1}{l_e}$$

$$m = \sin\theta = \frac{y_2 - y_1}{l_e}$$

$$\{q'\} = \{L\}\{q\}$$

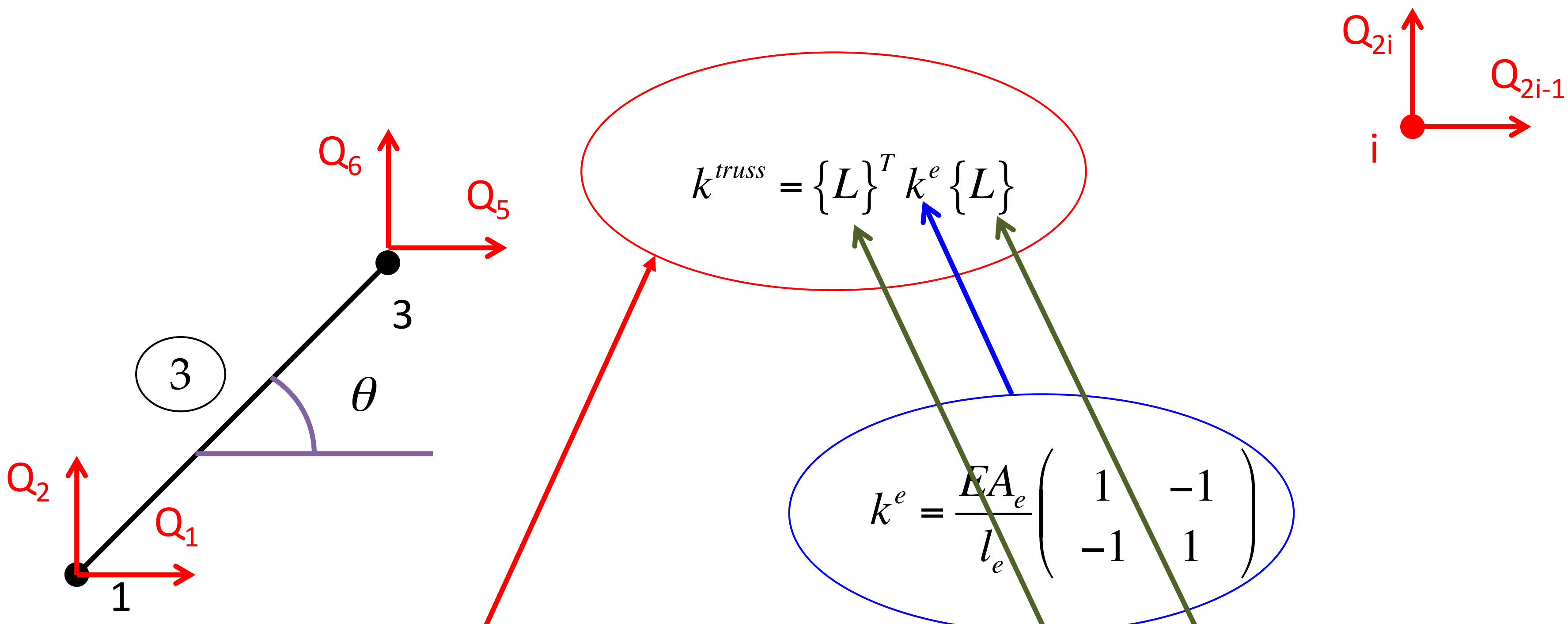
Element Stiffness



$$\{q'\} = \{L\}\{q\}$$

$$U_e = \frac{1}{2}\left[\{L\}\{q\}\right]^T k^e \{L\}\{q\} \Leftarrow U_e = \frac{1}{2}\{q'\}^T k^e \{q'\}$$

Element Stiffness



$$k^{truss} = \{L\}^T k^e \{L\}$$

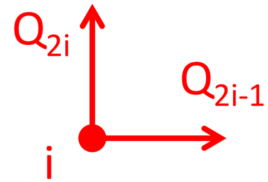
$$k^e = \frac{EA_e}{l_e}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

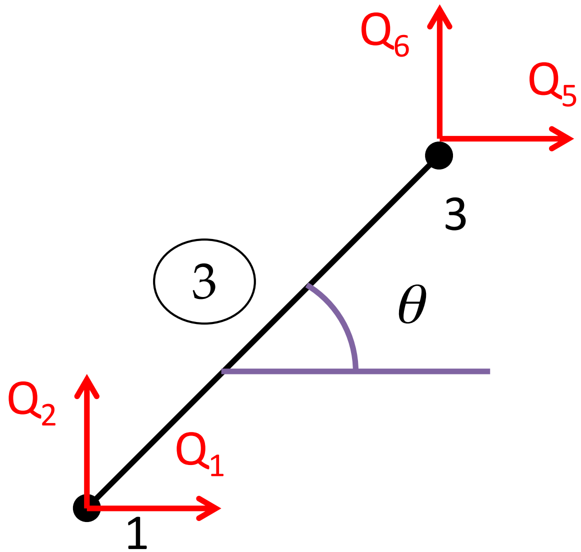
$$\{L\} = \begin{pmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{pmatrix}$$

$$U_e = \frac{1}{2}\left[\{L\}\{q\}\right]^T k^e \{L\}\{q\}$$

$$U_e = \frac{1}{2}\{q\}^T \{L\}^T k^e \{L\}\{q\}$$

Element Stiffness





$$k^{truss} = \{L\}^T k^e \{L\}$$

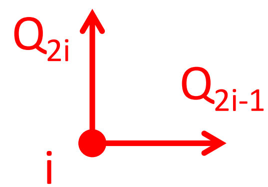
$$= \begin{pmatrix} l & 0 \\ m & 0 \\ 0 & l \\ 0 & m \end{pmatrix} \times \frac{EA_e}{l_e} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \times \begin{pmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{pmatrix}$$

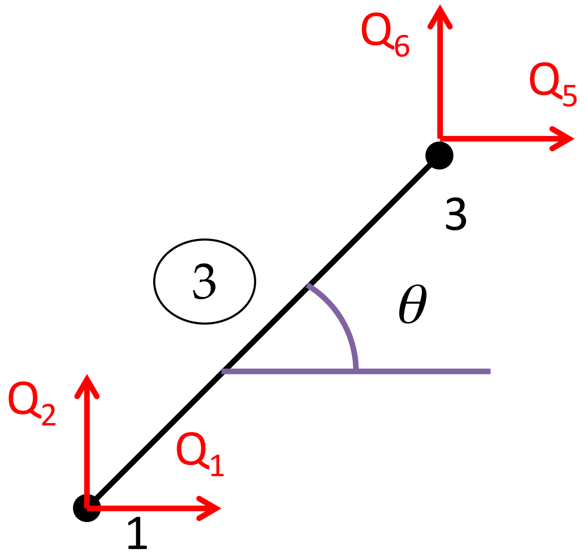
$$= \frac{EA_e}{l_e} \begin{pmatrix} l & 0 \\ m & 0 \\ 0 & l \\ 0 & m \end{pmatrix} \begin{pmatrix} l & m & -l & -m \\ -l & -m & l & m \end{pmatrix}$$

$$U_e = \frac{1}{2} \left[\{L\}\{q\} \right]^T k^e \{L\}\{q\}$$

$$U_e = \frac{1}{2} \{q\}^T \{L\}^T k^e \{L\}\{q\}$$

Element Stiffness





$$\therefore k^{truss} = \frac{EA_e}{l_e}\begin{pmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{pmatrix}$$

$$U_e = \frac{1}{2}\left[\{L\}\{q\}\right]^T k^e \{L\}\{q\}$$

$$U_e = \frac{1}{2}\{q\}^T \{L\}^T k^e \{L\}\{q\}$$

By the end of the notes:

You are expected to be able to:

- To execute coordinate transformations of 1-dimensional bar element for plane trusses
- To derive element stiffness matrix for plane trusses