LECTURE 2

MICROWAVE II

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By

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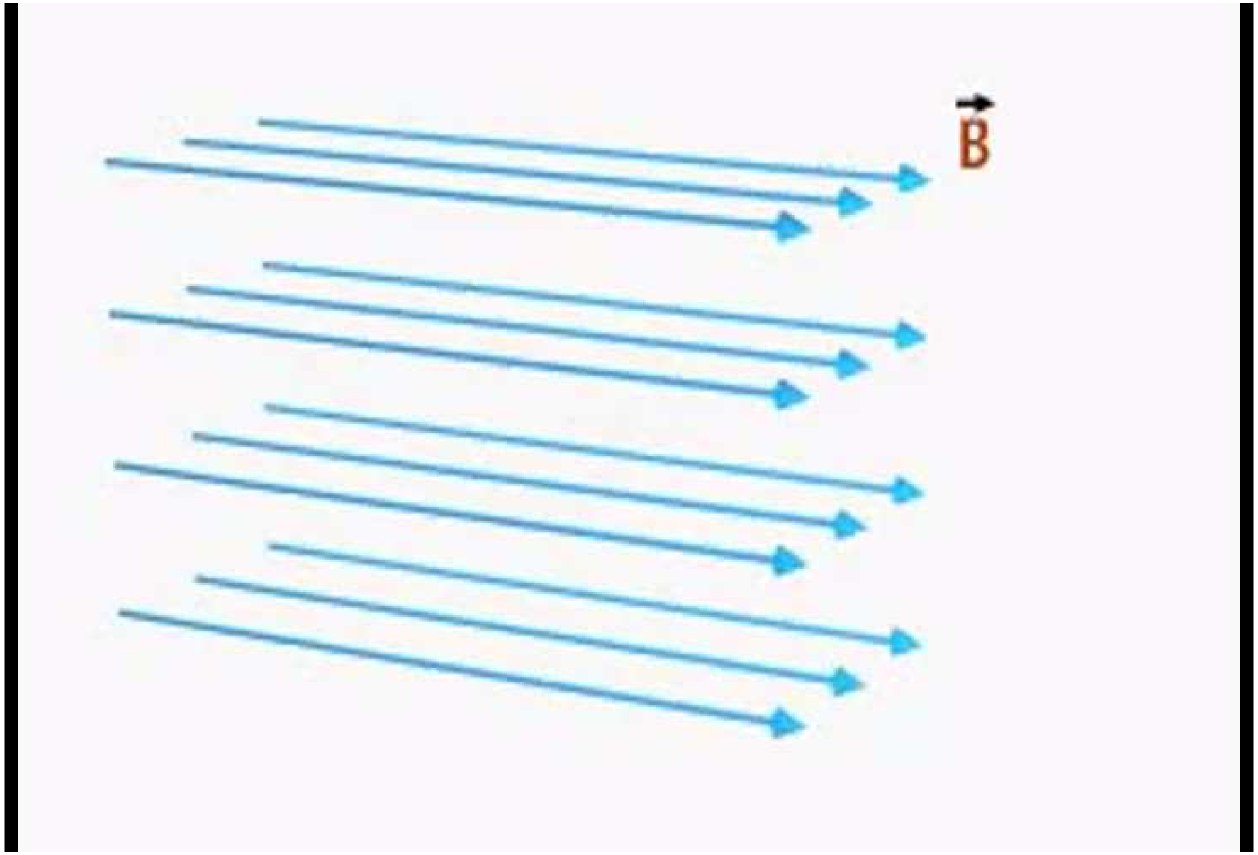
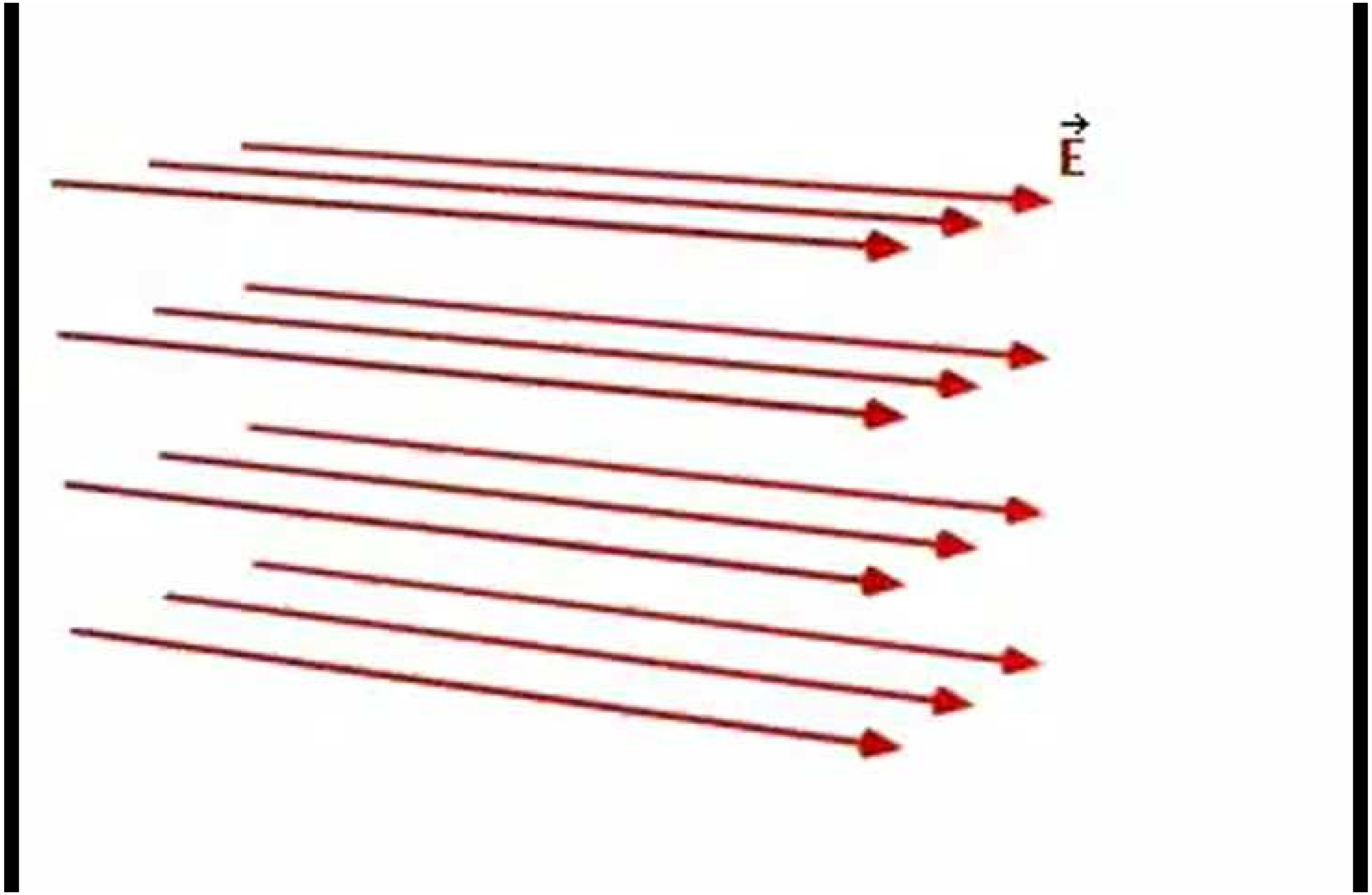
MAGNETIC FLUX



ELECTRIC FLUX



MAXWELL'S EQUATIONS

are four differential equations summarizing nature of electricity and magnetism: (formulated by James Clerk Maxwell around 1860):

$\vec{\nabla}\cdot\vec{D} = \rho$	Gauss's Law
$\vec{\nabla}\cdot\vec{B} = 0$	Gauss's Law for Magnetism
$\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$	Faraday's Law
$\vec{\nabla}\times\vec{H} = \vec{J} + \dfrac{\partial\vec{D}}{\partial t}$	Ampere's Law

MAXWELL'S EQUATIONS

(1) Electric charges generate electric fields.

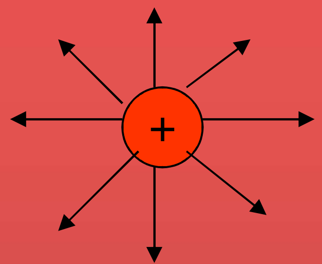
(2) Magnetic field lines are closed loops; there are no magnetic monopoles.

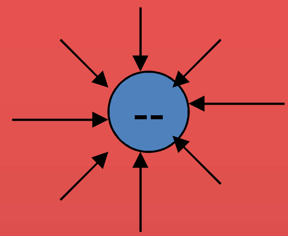
(3) Currents and changing electric fields produce magnetic fields.

(4) Changing magnetic fields produce electric fields.

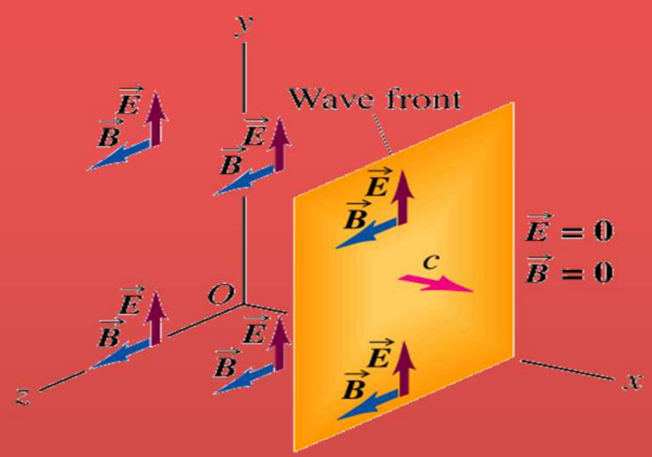
From Maxwell's equations one can derive another equation which has the form of a "*wave equation*".

Electric Charges and Fields





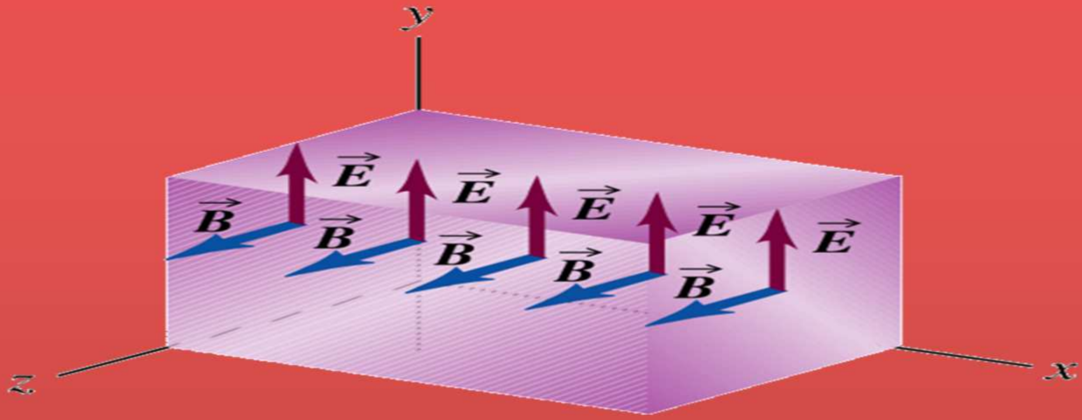
- When the charges oscillate, so do the the electric field lines which send out ripples.
- The ripples can be created in the directions orthogonal to the direction of oscillation (transverse wave).
- When a positive charge oscillates against a negative one, the ripples are loops of electric fields which propagate away from the charges.



An electromagnetic wave front.
The plane representing the wave front (yellow) moves to the right with speed c.

The **E** and **B** fields are uniform over the region behind the wave front but are zero everywhere in front of it.

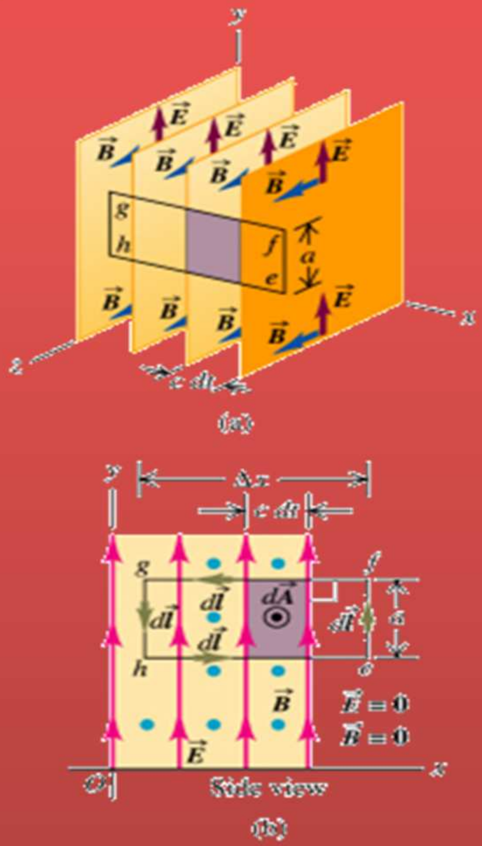
Gaussian surface for a plane electromagnetic wave.



The total electric flux and total magnetic flux through the surface are

both zero.

Faraday's law to a plane wave



$\int E \cdot dl = - d\Phi_B / dt$

1. $\int E \cdot dl = -Ea$ (cos 90° = 0)

2. In time *dt* the wave front moves to the right a distance *c dt*. The magnetic flux through the rectangle in the *xy*-plane increases by an amount $d\Phi_B$ equal to the flux through the shaded rectangle in the *xy*-plane with area ac dt, that is,

   $d\Phi_B = Bac\ dt$. So

   $-d\Phi_B / dt = -Bac$ and

   $E = Bc$

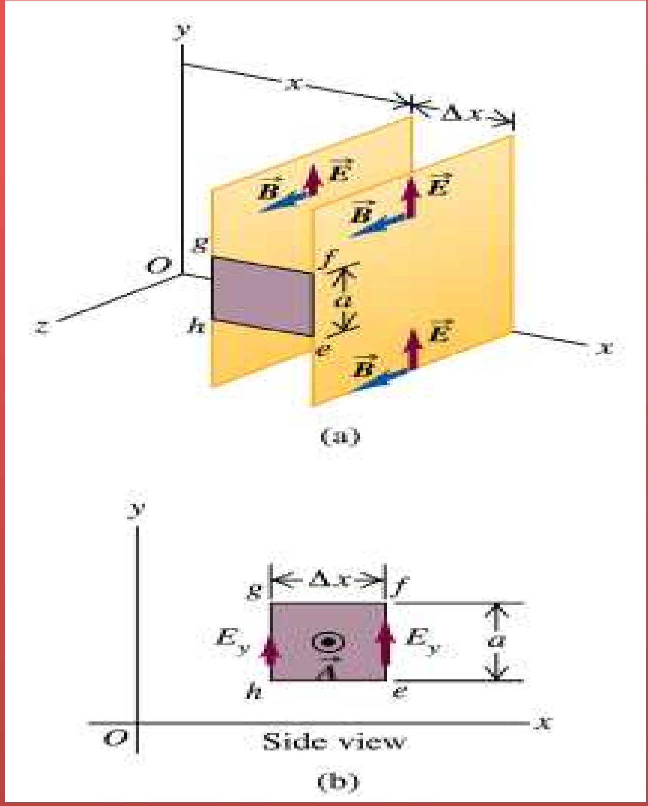
Faraday's law to a plane wave

Faraday's Law
 applied to a rectangle

with height *a* and

width D*x* parallel

to the
xy-plane.



Ampere's law to a plane wave

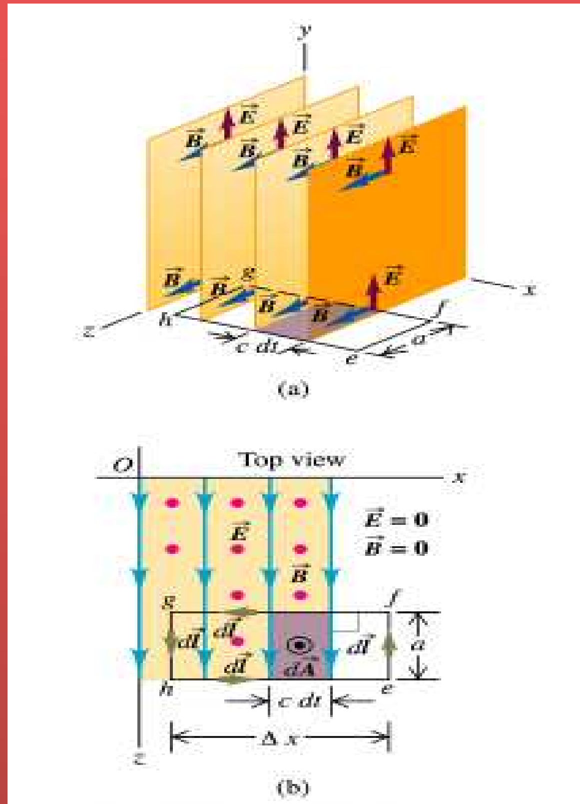
$\int B \cdot dl = \mu_o \varepsilon_o \, d\Phi_E / dt$

1. $\int B \circ dl = Ba$ (cos 90° = 0)

2. In time dt the wave front moves to the right a distance $c\,dt$. The electric flux through the rectangle in the xz-plane increases by an amount $d\,\Phi_E$ equal to E times the area $ac\,dt$ of the shaded rectangle, that is, $d\,\Phi_E = Eac\,dt$. Thus $d\,\Phi_E / dt = Eac$.
   $Ba = \mu_o \varepsilon_o Eac \rightarrow$ $B = \mu_o \varepsilon_o Ec$

and from $E = Bc$ and $B = \mu_o \varepsilon_o Ec$
We must have $c = 1 / (\mu_o \varepsilon_o)^{1/2}$

$c = 3.00\,(10)^8$ m/sec

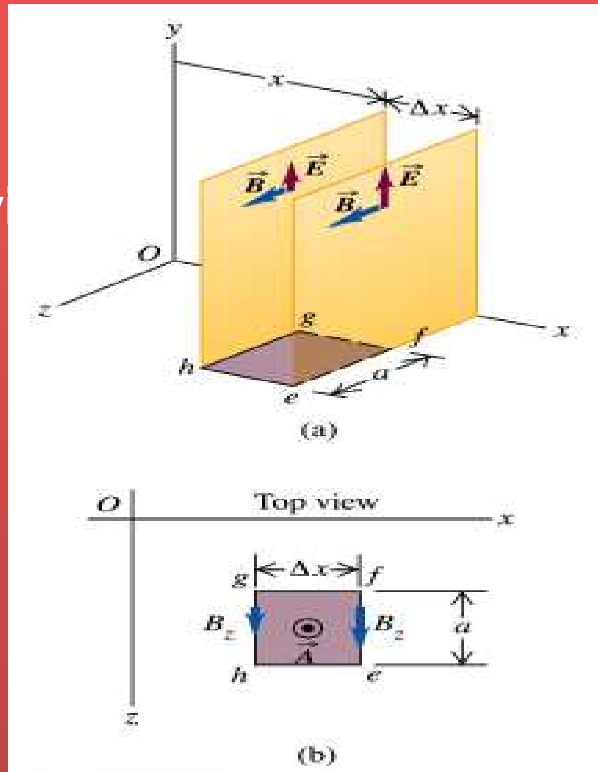


Ampere's law to a plane wave

Ampere's Law applied to a rectangle wi

height *a* and width

D*x* parallel to the *xz*-plane.



Guass Law

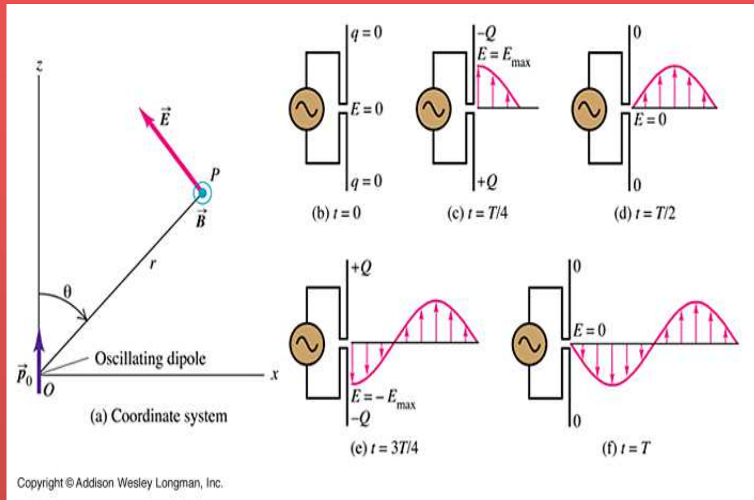
Gauss's law (electrical):

- The total electric flux through any closed surface equals the net charge inside that surface divided by ε_o
- This relates an electric field to the charge distribution that creates it

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_o}$$

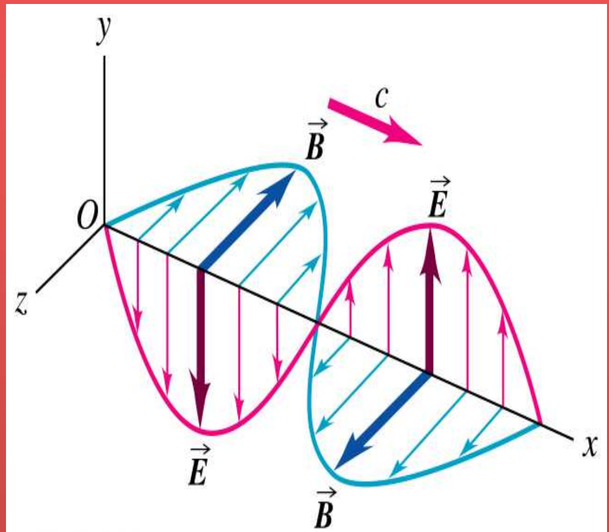
- Gauss's law (magnetism):
- The total magnetic flux through any closed surface is zero
- This says the number of field lines that enter a closed volume must equal the number that leave that volume
- This implies the magnetic field lines cannot begin or end at any point
- Isolated magnetic monopoles have not been observed in nature

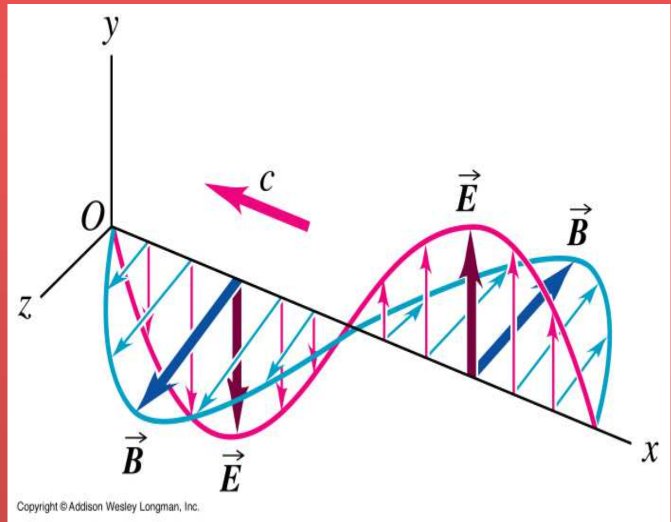
$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$$



•**One cycle in the production of an electro-magnetic wave by an oscillating electric dipole antenna.**

•**The red arrows represent the *E* field. (*B* not shown.)**





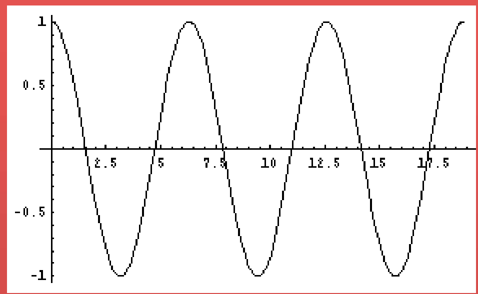
•Representation of the electric and magnetic fields in a propagating wave. One wavelength is shown at time *t* = 0.

•Propagation direction is *E* x *B*.

Harmonic Plane Waves

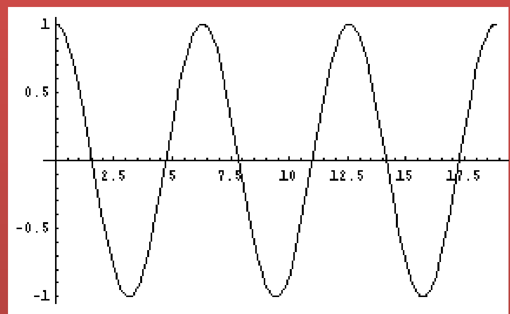
At t = 0

$\vec{E}$



At x = 0

$\vec{E}$



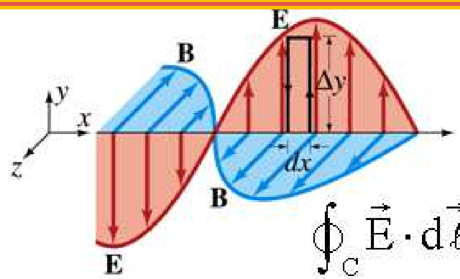
$\lambda =$ spatial period or wavelength

phase velocity

$$v = \frac{\lambda}{T} = f\lambda = \frac{2\pi}{T}\frac{\lambda}{2\pi} = \frac{\omega}{k}$$

$T =$ temporal period

Applying Faraday to radiation



$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

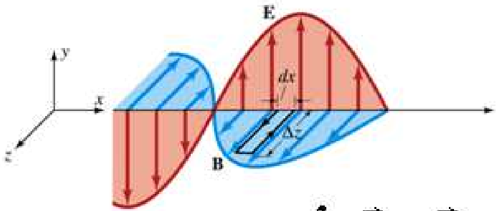
$$\oint_C \vec{E} \cdot d\vec{\ell} = \left(E + dE\right)\Delta y - E\Delta y = dE\Delta y$$

$$\frac{d\Phi_B}{dt} = \frac{dB}{dt} dx\Delta y$$

$$dE\Delta y = -\frac{dB}{dt} dx\Delta y$$

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

Applying Ampere to radiation



$$\oint_C \vec{B}\cdot d\vec{\ell} = \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

$$\oint_C \vec{B}\cdot d\vec{\ell} = B\Delta z - \left(B + dB\right)\Delta z = -dB\Delta z$$

$$\frac{d\Phi_E}{dt} = \frac{dE}{dt}\,dx\Delta z$$

$$-dB\Delta z = \mu_o \varepsilon_o \frac{dE}{dt}\,dx\Delta z$$

$$\frac{dB}{dx} = -\mu_o \varepsilon_o \frac{dE}{dt}$$

Fields are functions of both position (x) and time (t)

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

$$\frac{dB}{dx} = -\mu_\circ \varepsilon_\circ \frac{dE}{dt}$$

Partial derivatives are appropriate

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial B}{\partial x} = -\mu_\circ \varepsilon_\circ \frac{\partial E}{\partial t}$$

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x}\frac{\partial B}{\partial t}$$

$$\frac{\partial}{\partial t}\frac{\partial B}{\partial x} = -\mu_\circ \varepsilon_\circ \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial x^2} = \mu_\circ \varepsilon_\circ \frac{\partial^2 E}{\partial t^2}$$

This is a wave equation!

The Trial Solution

The simplest solution to the partial differential equations is a sinusoidal wave:

$E = E_{max} \cos (kx - \omega t)$

$B = B_{max} \cos (kx - \omega t)$

The angular wave number is $k = 2\pi/\lambda$

λ is the wavelength

The angular frequency is $\omega = 2\pi f$

f is the wave frequency

The Trial Solution

$$E = E_y = E_o \sin(kx - \omega t)$$

$$\frac{\partial^2 E}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial x^2} = -k^2 E_o \sin(kx - \omega t) \qquad \frac{\partial^2 E}{\partial t^2} = -\omega^2 E_o \sin(kx - \omega t)$$

$$-k^2 E_o \sin(kx - \omega t) = -\mu_o \varepsilon_o \omega^2 E_o \sin(kx - \omega t)$$

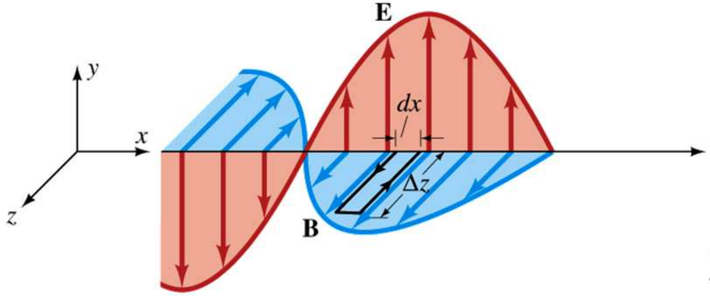
$$\frac{\omega^2}{k^2} = \frac{1}{\mu_o \varepsilon_o}$$

The speed of light

$$v = \frac{\lambda}{T} = f\lambda = \frac{2\pi}{T}\frac{\lambda}{2\pi} = \frac{\omega}{k}$$

$$v = c = \frac{\omega}{k} = \sqrt{\frac{1}{\mu_o \varepsilon_o}}$$

Another look



$$\frac{dE}{dx} = -\frac{dB}{dt}$$

$$B = B_z = B_o \sin(kx - \omega t) \qquad E = E_y = E_o \sin(kx - \omega t)$$

$$\frac{d}{dx} E_o \sin(kx - \omega t) = -\frac{d}{dt} B_o \sin(kx - \omega t)$$

$$E_o k \cos(kx - \omega t) = B_o \omega \cos(kx - \omega t)$$

$$\frac{E_o}{B_o} = \frac{\omega}{k} = c = \sqrt{\frac{1}{\mu_o \varepsilon_o}}$$