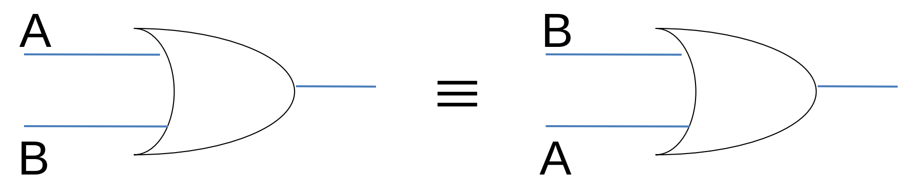
Logic Gates and Boolean Algebra

- Logic Gates
  - Inverter, OR, AND, Buffer, NOR, NAND, XOR, XNOR
- Boolean Theorem
  - Commutative, Associative, Distributive Laws
  - Basic Rules
- DeMorgan's Theorem
- Universal Gates
  - NAND and NOR
- Canonical/Standard Forms of Logic
  - Sum of Product (SOP)
  - Product of Sum (POS)
  - Minterm and Maxterm

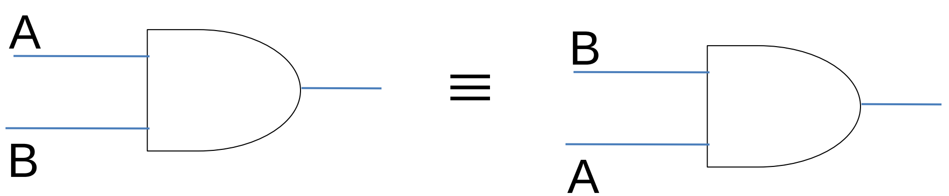
Boolean Theorem

- Commutative Law
  - In terms of the result, the order in which variables are ORed or ANDed makes no difference.

$$\boxed{A + B = B + A}$$



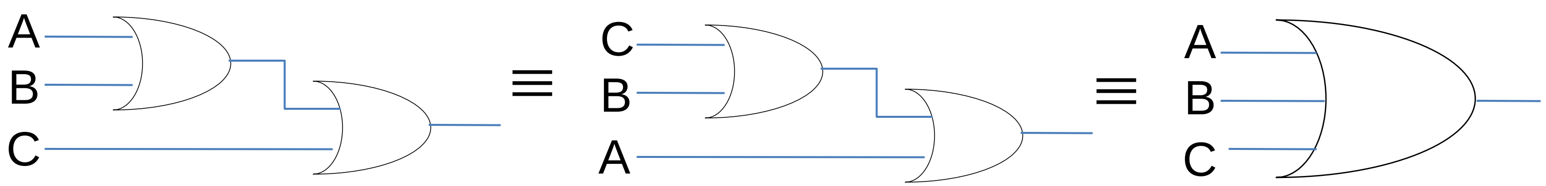
$$\boxed{AB = BA}$$



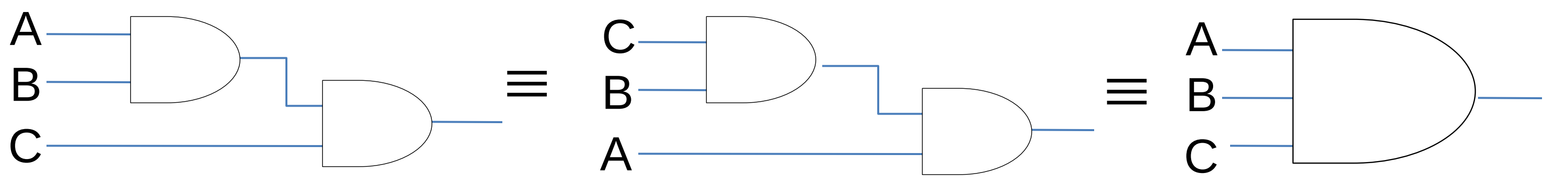
Boolean Theorem

- Associative Law
  - When ORing or ANDing more than two variables, the result is the same regardless of the grouping of the variables.

$$A + (B + C) = (A + B) + C$$



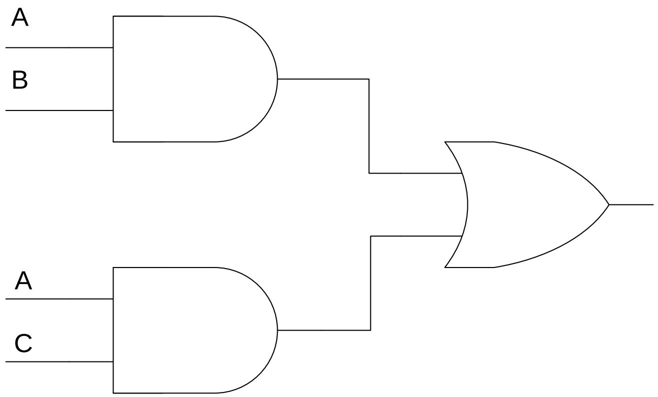
$$A(BC) = (AB)C$$



Boolean Theorem

- Distributive Law
  - A common variable can be factored from an expression just as in ordinary algebra.

$$\boxed{AB + AC = A(B + C)}$$



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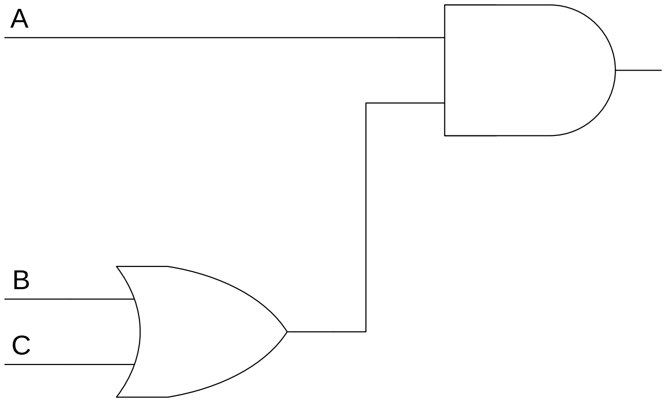


Boolean Theorem

- Basic Rules

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

Boolean Simplification - Example

- Using boolean theorem, Simplify the expression:

$$AB + A(B+C) + B(B+C)$$

Apply distributive law,

$$AB + AB + AC + BB + BC$$

Apply rule 7 (BB = B), and rule 5 (AB + AB = AB)

$$AB + AC + B + BC$$

Apply rule 10 (B + BC = B)

$$AB + AC + B$$

Boolean Simplification - Example

$$AB + AC + B$$

Apply rule 10 (AB + B = B)

$$B + AC$$

At this point, the expression is simplified as much as possible

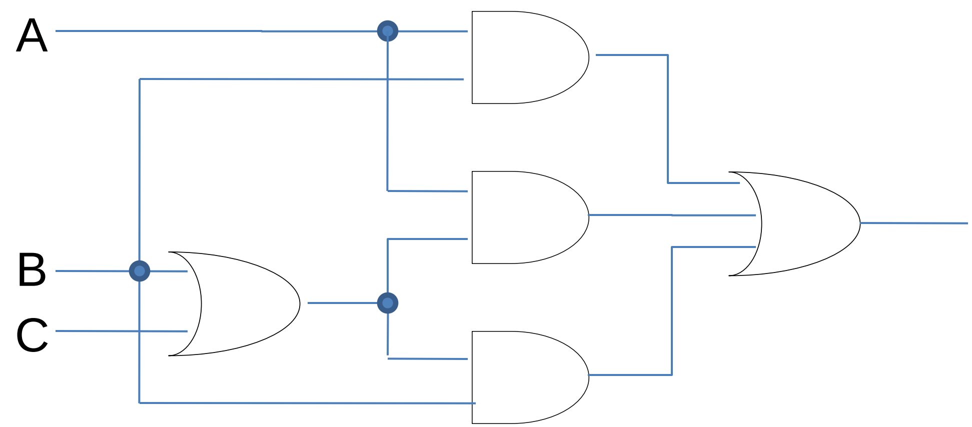
Original expression is $AB + A(B + C) + B(B + C)$

Which is logically equal to $B + AC$

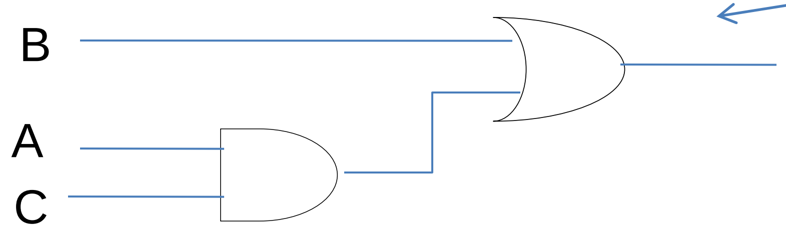
what is the advantage of Boolean simplification?

Boolean Simplification - Example

Original expression is $AB + A(B+C) + B(B+C)$



Which is logically equal to $B + AC$



Faster
Compact design
Lower cost

Boolean Simplification - Example

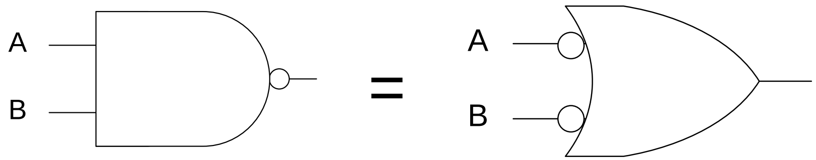
- Applying boolean theorem for logic simplification depends on a thorough knowledge of boolean algebra, with some ingenuity and cleverness
- Please look at Floyd's book examples 4-10, 4-11, and 4-12, as well as some exercises in the book to gain experience

DeMorgan’s Theorem

- The complement of a product of variables is equal to the sum of the complemented variables

Theorem 1

$$\overline{AB} = \overline{A} + \overline{B}$$

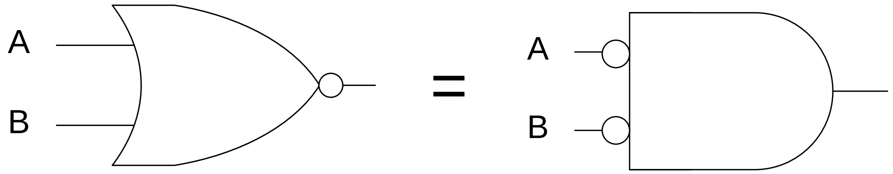


A	B	$\overline{A \cdot B}$	$\overline{A} + \overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

DeMorgan's Theorem

Theorem 2

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$



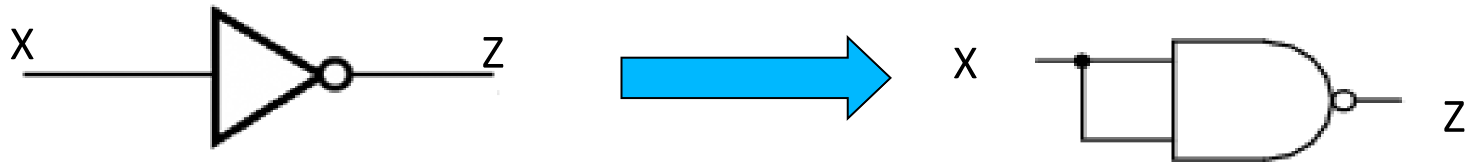
A	B	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Universal Gates

- NAND and NOR gates are known as Universal gates because all logic gates can be represented by NAND and NOR
- NAND and NOR are the cheapest and smallest to manufacture in Integrated Circuits compared to AND and OR
- Therefore NAND and NOR are always used in practical circuit design

NAND Universal Gates

- How to represent inverter using NAND gates?



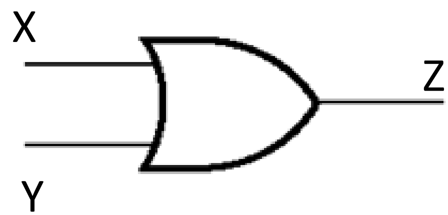
NAND gate truth table

X	Y	$Z = \overline{X \cdot Y}$
0	0	1
~~0~~	~~1~~	~~1~~
~~1~~	~~0~~	~~1~~
1	1	0

Short the inputs together

NAND Universal Gates

- How to represent OR gate using NAND gates?

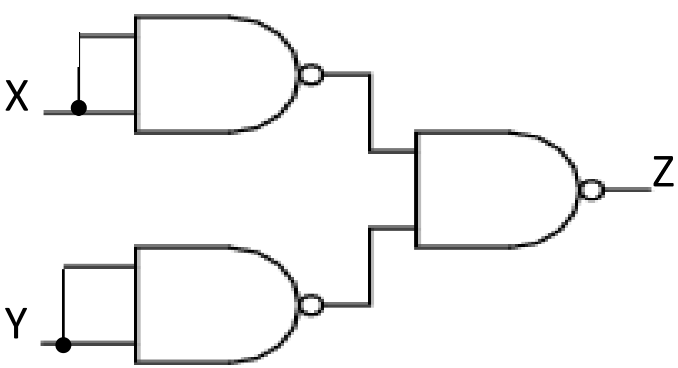


Logic expression for OR gate:

$$X + Y \equiv \overline{\overline{X + Y}}$$

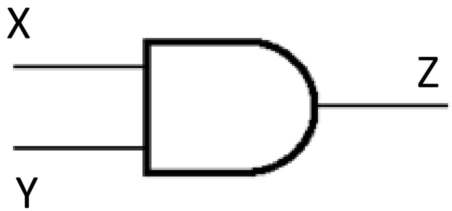
Using DeMorgan's Theorem,

$$\overline{\overline{X} \cdot \overline{Y}}$$



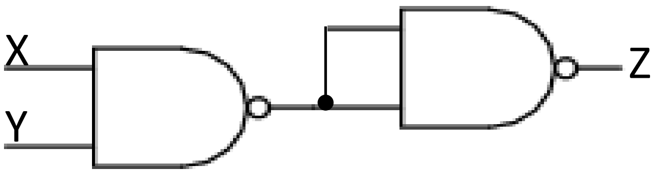
NAND Universal Gates

- How to represent AND gate using NAND gates?



Logic expression for AND gate:

$$X \cdot Y \quad \equiv \overline{\overline{X \cdot Y}}$$



NOR Universal Gates