# SEE1223: Digital Electronics

## 3 – Combinational Logic Design

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# Karnaugh Maps (K-Map)

- K-Map structure
  - 2,3, and 4-variable Karnaugh Maps
- K-Map Grouping and Logic Simplification
- K-Map SOP and POS terms
- K-Map Don't Care Conditions
- Logic Design using K-Maps

# Introduction

- Karnaugh Map (K-Map) is a tool for simplifying digital logic with 2-6 variables
- K-Map, if properly used will produce the simplest SOP and POS expression possible, known as the *minimum expression*
- K-Map simplifies logic through SOP and POS boolean expressions, and truth table
- In this class, we'll look at logic simplification of 2, 3, and 4 variables

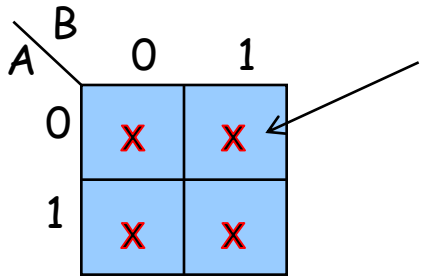
# 2-variable K-Maps

K-Map is a representation of a truth table, but can be used to obtain Boolean expressions

F(A,B) truth table

| A B | F |
|---|---|
| 0 0 | x |
| 0 1 | x |
| 1 0 | x |
| 1 1 | x |

2-variable K-Map




Cell – location of each entry

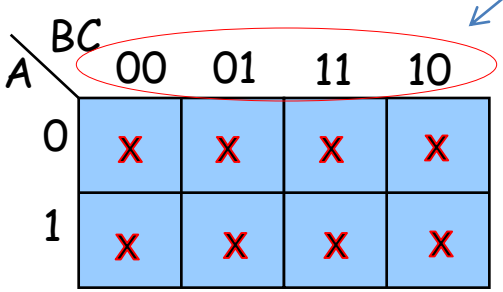
# 3-variable K-Map

F(A,B,C) truth table

| A B C | F |
|---|---|
| 0 0 0 | x |
| 0 0 1 | x |
| 0 1 0 | x |
| 0 1 1 | x |
| 1 0 0 | x |
| 1 0 1 | x |
| 1 1 0 | x |
| 1 1 1 | x |

3-variable K-Map

Gray code ordering



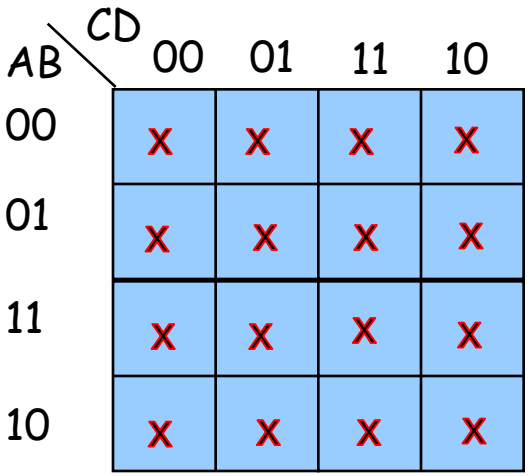
# 4-variable K-Map

F(A,B,C,D) truth table

| A B C D | F |
|---|---|
| 0 0 0 0 | x |
| 0 0 0 1 | x |
| 0 0 1 0 | x |
| 0 0 1 1 | x |
| 0 1 0 0 | x |
| 0 1 0 1 | x |
| 0 1 1 0 | x |
| 0 1 1 1 | x |

| A B C D | F |
|---|---|
| 1 0 0 0 | x |
| 1 0 0 1 | x |
| 1 0 1 0 | x |
| 1 0 1 1 | x |
| 1 1 0 0 | x |
| 1 1 0 1 | x |
| 1 1 1 0 | x |
| 1 1 1 1 | x |

4-variable K-Map



# K-Map Example

- Given the following standard form of SOP, complete the truth table and K-map

$$F = \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$

| A B C | F | |
|---|---|---|
| 0 0 0 | 0 | |
| 0 0 1 | 1 | $\overline{A}\,\overline{B}C$ |
| 0 1 0 | 1 | $\overline{A}B\overline{C}$ |
| 0 1 1 | 0 | |
| 1 0 0 | 0 | |
| 1 0 1 | 0 | |
| 1 1 0 | 1 | $AB\overline{C}$ |
| 1 1 1 | 1 | $ABC$ |

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |

# K-Map Example

- Given the following SOP expression, complete the K-Map

$$F = \overline{B}\overline{C} + A\overline{B} + AB\overline{C} + A\overline{B}C\overline{D} + \overline{A}\overline{B}CD + A\overline{B}CD$$

*F = 1*

when B = 0 and C = 0 (1st minterm)
when A = 1 and B = 0 (2nd minterm)
when A = 1, B = 1, and C = 0 (3rd minterm)
when A = 1, B = 0, C = 1, and D = 0 (4th minterm)
when A = 0, B = 0, C = 0, and D = 1 (5th minterm)
when A = 1, B = 0, C = 1, and D = 1 (6th minterm)

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 0 | 0 |
| 01 | 0 | 0 | 0 | 0 |
| 11 | 1 | 1 | 0 | 0 |
| 10 | 1 | 1 | 1 | 1 |

# K-Map Grouping

- After SOP expression has been mapped, minimum expression is obtained by grouping the 1's and determining the minimum SOP expression from the map
- When grouping the 1's, the goal is to maximize the size of the groups, and minimize the number of groups

# K-Map Grouping (cont.)

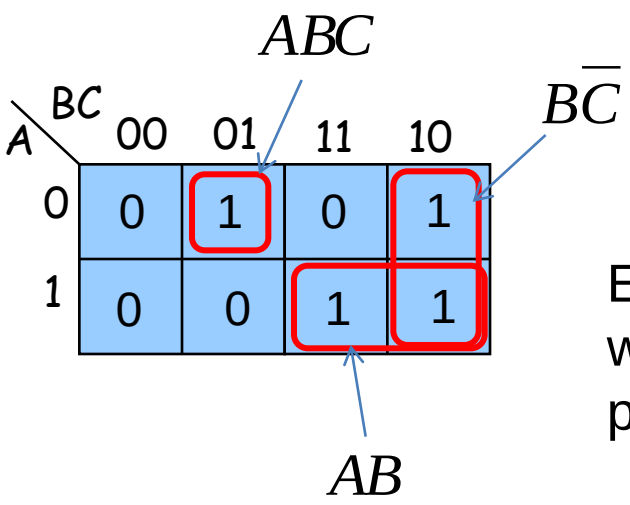
- Rules for grouping of 1's
  - A group must contain either 1, 2, 4, 8, or 16 cells. For x-variable K-map, $2^x$ cells is maximum
  - Each cell in a group must be adjacent to one or more cells in that same group, but all cells in the group don't have to be adjacent to each other
  - Always include the largest possible number of 1's in a group
  - Each 1 on the map must be included in at least one group. The 1's already in a group can be included in another group as long as the overlapping groups include common 1's

# K-Map Minimum Product Term

- For 3-variable K-Map
  - 1 cell group yields a 3-variable product term
  - 2 cell group yields a 2-variable product term
  - 4 cell group yields a 1-variable product term
  - 8 cell group yields a value of 1 for the expression
- For 4-variable K-Map
  - 1 cell group yields a 4-variable product term
  - 2 cell group yields a 3-variable product term
  - 4 cell group yields a 2-variable product term
  - 8 cell group yields a 1-variable product term
  - 16- cell group yields a value of 1 for the expression

# K-Map Simplification

- Group the 1's and find the minimum SOP expression in the K-Map below




Expression is minimized when taking large cell possible

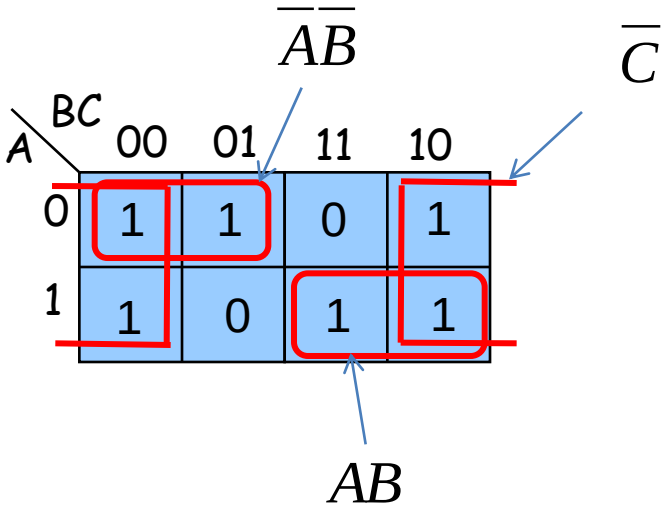
$$F = AB + \overline{A}\overline{B}C + B\overline{C}$$

What is the SOP expression if each cell is taken as a group?

$$F = \overline{A}\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$

# K-Map Simplification

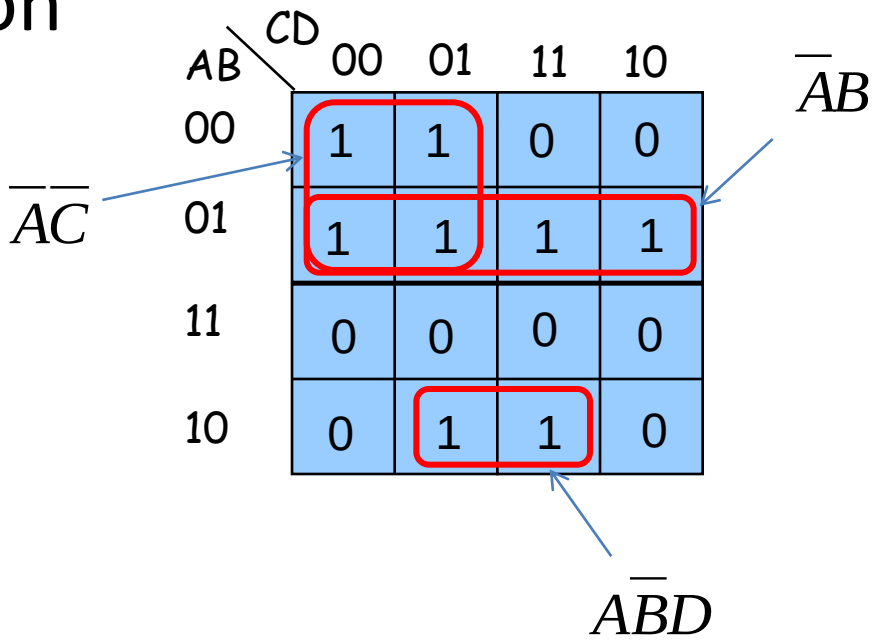
- Find the minimum SOP expression for the logic expression: $F(A,B,C) = \prod(3,5)$




$$F = AB + \overline{A}\,\overline{B} + \overline{C}$$

# K-Map Simplification

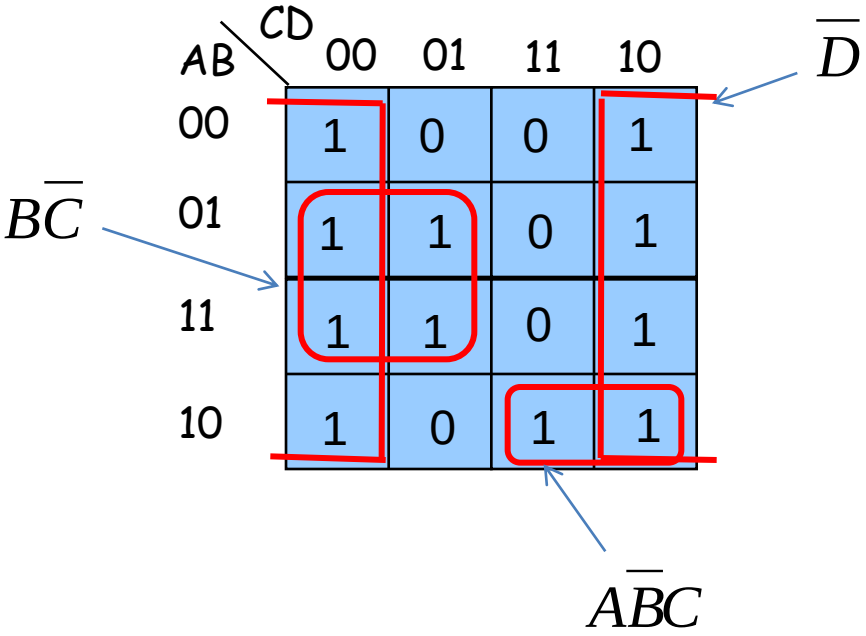
- Group the 1's and find the minimum SOP expression




$$F = \overline{A}B + \overline{A}\overline{C} + A\overline{B}D$$

# K-Map Simplification

- Find the minimum expression for the logic expression: $F(A,B,C,D) = \sum(0,2,4,5,6,8,10,11,12,13,14)$



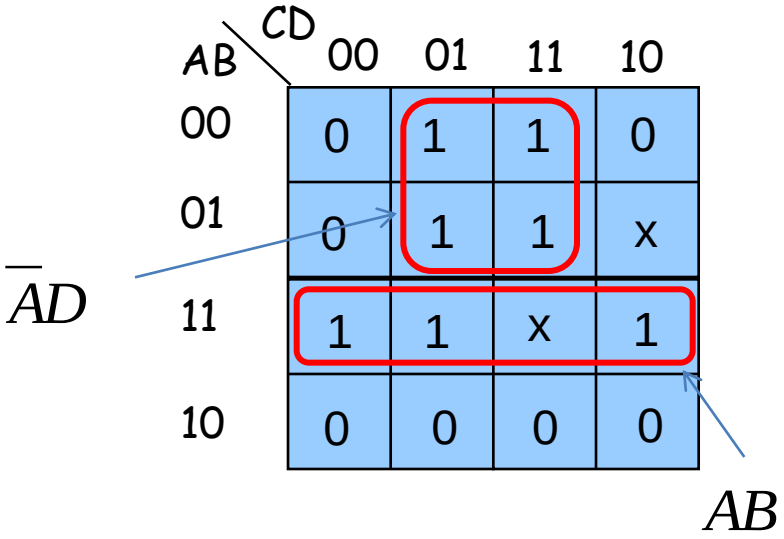

$$F = \overline{D} + B\overline{C} + A\overline{B}C$$

# Don’t Care Conditions

- Don’t Care is the condition when the output can either be ‘1’ or ‘0,’ which is denoted by ‘x’ in the truth table or K-Map
- For both SOP and POS minimum expression, ‘x’ can be included or ignored

# Don't Care Condition (cont.)

- Find minimum SOP expression for the following K-Map




$$F = \overline{A}D + AB$$

If the 'x' is replaced by '0,' find the minimum SOP expression

$$F = \overline{A}D + AB\overline{C} + AB\overline{D}$$